Superkicks in ultrarelativistic encounters of spinning black holes

Ulrich Sperhake,[1,2,3] Emanuele Berti,[3,2] Vitor Cardoso,[4,3] Frans Pretorius,[5] and Nicolas Yunes[5,6,7]

[1]*Institut de Ciències de l'Espai (CSIC-IEEC), Facultat de Ciències, Campus UAB, E-08193 Bellaterra, Spain*
[2]*California Institute of Technology, Pasadena, California 91125, USA*
[3]*Department of Physics and Astronomy, The University of Mississippi, University, Mississippi 38677, USA*
[4]*CENTRA, Departamento de Física, Instituto Superior Técnico, Universidade Técnica de Lisboa-UTL, Av. Rovisco Pais 1, 1049 Lisboa, Portugal*
[5]*Department of Physics, Princeton University, Princeton, New Jersey 08544, USA*
[6]*Department of Physics and MIT Kavli Institute, 77 Massachusetts Avenue, Cambridge, Massachusetts 02139, USA*
[7]*Harvard-Smithsonian Center for Astrophysics, 60 Garden Street, Cambridge, Massachusetts 02138, USA*
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We study ultrarelativistic encounters of two spinning, equal-mass black holes through simulations in full numerical relativity. Two initial data sequences are studied in detail: one that leads to scattering and one that leads to a grazing collision and merger. In all cases, the initial black hole spins lie in the orbital plane, a configuration that leads to the so-called *superkicks*. In astrophysical, quasicircular inspirals, such kicks can be as large as ~3000 km/s; here, we find configurations that exceed ~15 000 km/s. We find that the maximum recoil is to a good approximation proportional to the total amount of energy radiated in gravitational waves, but largely *independent* of whether a merger occurs or not. This shows that the mechanism predominantly responsible for the superkick is not related to merger dynamics. Rather, a consistent explanation is that the ''bobbing'' motion of the orbit causes an asymmetric beaming of the radiation produced by the in-plane orbital motion of the binary, and the net asymmetry is balanced by a recoil. We use our results to formulate some conjectures on the ultimate kick achievable in any black hole encounter.

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I. INTRODUCTION

One of the more interesting consequences of binary coalescence is the *recoil* or *kick* velocity that the center of mass can acquire during the event. This possibility was first discussed by Bekenstein [1]. Kicks are generated by an asymmetry in the momentum carried away by gravitational waves (GWs): if more momentum is carried away in any one direction, then the center of mass will ''react'' by acquiring a velocity in the opposite direction to conserve momentum.

In one particularly interesting scenario where the black hole (BH) spins are equal in magnitude, opposite in direction, yet within the orbital plane, the recoil velocity can become quite large, a phenomenon that is sometimes called a *superkick* [2–5]. At first glance, it is somewhat surprising that this configuration can lead to such a large recoil, as this is a highly symmetric orbit: the masses are equal, the spins are antialigned, and the system's total angular momentum equals the orbital angular momentum. Furthermore, the resultant kick velocity depends sinusoidally on the initial phase of the binary, and linearly (at leading order) on the magnitude of the individual BH spins.

A schematic explanation of the superkick was initially offered in Ref. [6], as being due to the ''dragging of the inertial frame'' of one BH relative to the other, and vice versa. This was expanded upon in [7,8], where it was pointed out that, in addition to the frame-dragging effect, there is also a spin-curvature coupling effect responsible for the superkick at the same post-Newtonian order. From a distant observer's perspective, these effects cause the orbital plane to ''bob'' up and down in a sinusoidal manner, while the binary inspirals, with frequency equal to the orbital frequency. This bobbing motion by itself does not directly produce the radiation that must be balanced by a recoil. Rather, the bobbing causes the radiation produced by the binary's in-plane orbital motion to be blue- or red-shifted, in synchrony with the bobbing. It is this asymmetry in the radiation pattern that ultimately results in net linear momentum radiated in a direction orthogonal to the orbital plane, balanced by the remnant BH moving in the opposite direction, after coalescence.

More recently, however, Gralla *et al.* [9] have argued, based on an electromagnetic analogue model, that the bobbing motion is purely ''kinematical'' in nature, and not responsible for the recoil. Rather, they speculate that the recoil has to arise in a process directly related to the merger event which causes field momentum to be ''released'' and radiated to infinity. Here we present a first study of the ultrarelativistic scattering of two BHs in the superkick configuration, in part to address the issue about the origin of the superkick, and in part to continue our study of the high-energy regime in BH collisions. Specifically, we study two families of initial data, one leading to merger and one leading to scattering, although in the latter the BHs interact strongly. In *both* cases we find essentially *identical* recoil behavior of the center of mass following the interaction: the recoil direction is orthogonal to the orbital plane, and the magnitude varies sinusoidally with the initial phase, with a maximum proportional to the *net* energy radiated in GWs. This is completely consistent with the original scenario where bobbing-induced blue- or redshifting of the radiated energy leads to the recoil. The only effect of the merger is to slightly enhance the radiated energy, and hence the maximum recoil.

These conclusions do not necessarily imply that the electromagnetic analogue in [9] is incorrect. However, since radiation-reaction effects were not included in that study, it is conceivable that the same phenomena would arise in the scattering of appropriately aligned magnetic dipoles. In fact, then the two explanations described above might more be a difference in semantics, i.e., an issue of whether one considers radiation to be a "release" of "field momentum," which could happen regardless of merger, as it does in the BH scattering case.

Several other interesting conclusions can be drawn from the results of this study, other than implications for the nature of the mechanism of the superkick. First, in the scattering cases, we also see situations where a so-called *antikick* is present, i.e., where the maximum instantaneous net linear momentum radiated is not equal to the final value. Thus, again, explanations of this phenomenon relying on effects due to the presence of a common horizon (as in Ref. [10]) cannot be the complete picture. Second, to gauge the effect that spin in this configuration has on the overall energy and *angular momentum* radiated in a merger, we compare the results obtained for each sequence with those for an equivalent binary, except the BHs initially have zero spin. We find that spin has very little effect on the radiated energy and angular momentum. Third, we show that subdominant effects in the spins scale as predicted by the "spin expansion formalism" developed by Boyle, Kesden, and Nissanke [11,12]. This is true for both merging and scattering configurations; the latter result is nontrivial, since the spin expansion was explicitly formulated for binaries that lead to mergers. Fourth and last, these simulations result in the largest superkicks seen to-date in merger simulations, upwards of $\sim 15,000$ km/s. This is a factor of 5 larger than the maximum yet seen in quasicircular BH coalescences, and 50% larger than those obtained by Healy *et al.* in hyperbolic encounters [13]. In Sec. III C we provide some speculations on the maximum kick that could theoretically be achievable in any ultrarelativistic encounter. We note that such enormous superkicks are not expected to occur in realistic astrophysical scenarios.

An outline of the rest of the paper is as follows. Section II discusses the numerical implementation of the problem and related numerical uncertainties. In Sec. III we present our results on the radiated energy and linear momentum. We conclude in Sec. IV with a summary of our findings. Throughout this work we use geometrical units ($G = c = 1$), unless otherwise noted.

II. NUMERICAL SIMULATIONS

We have performed numerical simulations with the LEAN code [14] which evolves the Einstein equations using the Baumgarte-Shapiro-Shibata-Nakamura (BSSN) formulation [15,16] in combination with the moving puncture method [17,18]. The exact form of our evolution system is given by Eqs. (11), (A1), (A4), and (A6–A8) in Ref. [14]. For evolving the shift β^i, we follow [19] and employ a first order in time version of the so-called "Gamma driver" [see their Eq. (26)]. The free parameter η is set to $\eta = 0.5$ (in code units) in all simulations. This corresponds to $M\eta = 0.868$, where M is the total center-of-mass energy of the system.

The LEAN code is based on the CACTUS computational toolkit [20] and uses mesh refinement provided by CARPET [21,22]. Initial data are calculated according to the puncture method [23] with Bowen-York parameters [24] using the CACTUS thorn TWOPUNCTURES based on Ansorg's spectral solver [25]. Apparent horizons are located and analyzed with Thornburg's AHFINDERDIRECT [26,27]. GWs are extracted using the Newman-Penrose scalar Ψ_4, as summarized in Appendix C of Ref. [14]. The energy, linear and angular momentum carried by GWs are obtained from Ψ_4 according to Eqs. (2.8), (2.11), and (2.24) of Ref. [28]. For more details on the code we refer the reader to Refs. [14,29].

A. Initial configurations

All of our simulations are performed in the center-of-mass frame of the binary, defined as the frame with zero Arnowitt-Deser-Misner (ADM) linear momentum [30]. We determine the initial parameters of each BH under the assumption of isolated horizons. This approximation is justified by the large initial separations used for all simulations. We thus obtain the irreducible mass $M_{\mathrm{irr},i}$ for BH $i = 1$ and 2 and calculate the BH rest mass from Christodoulou's [31] relation

$$M_i^2 = M_{\mathrm{irr},i}^2 + \frac{S_i^2}{4M_{\mathrm{irr},i}^2}, \tag{1}$$

where S_i is the spin magnitude of the ith BH. The boost parameter is defined by the ratio of dynamic to rest mass $\gamma = M_{\mathrm{dyn}}/(M_1 + M_2)$, where

$$M_{\mathrm{dyn}}^2 = M_1^2 + P_1^2 + M_2^2 + P_2^2, \tag{2}$$

and P_i is the magnitude of either BH's initial linear momentum. In this work, we consider equal-mass binaries so that the boost of the individual BHs equals γ. In practice, both BHs start on the x-axis at location $\pm x_0$ and their initial Bowen-York momenta are $\mathbf{P} = (\mp P_x, \pm P_y, 0)$, so that the initial orbital angular momentum is given by $L = dP_y = 2x_0 P_y$.

With these definitions, we can characterize a binary initial configuration using the following parameters: the

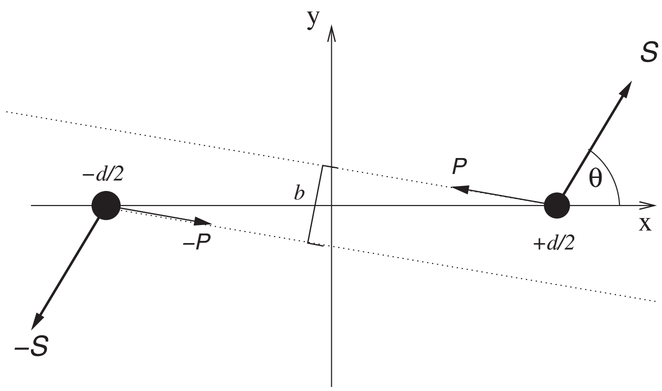

FIG. 1. Illustration of the BH binary initial configuration.

boost parameter γ, the magnitude of the dimensionless spin $\chi_i = S_i/M_i^2$ (where in all of our simulations $\chi_1 = \chi_2 = \chi$), the initial separation d, the impact parameter $b = L/P$, and the orientation of the spins measured by the angle θ relative to the coordinate axis connecting the initial BH positions (see Fig. 1).

For both sequences, we fix the boost parameter $\gamma = 1.52$, corresponding to $P/M = 0.374$, the dimensionless spin $\chi = 0.621$, and the initial separation $d = 58.2M$. The two sequences differ in the impact parameter; $b = 3.34M$ for the s-sequence (scattering) and $b = 3.25M$ for the m-sequence (merging binaries). We carried out a total of 20 simulations for selected values of the angle θ in the range [0°, 360°]. For comparison, we also present results from two nonspinning, equal-mass binaries with the same rest mass, boost, and impact parameters. Radiated energy and angular momenta, and (for the merger cases) final horizon properties are summarized in Table I (some of these quantities have not yet been introduced, but they will be defined later on in the paper).

TABLE I. Initial and final parameters for the two sequences of binary models. Note that in all cases the estimated uncertainties in these quantities (not shown) from numerical truncation error or finite extraction radius is *larger* than the intrinsic variation within each sequence, *including* the two nonspinning comparison cases. Therefore, rather than list the values for all the separate cases, here we just list the average value, and the maximum deviation relative to the average. Note that for merger cases we only have apparent horizon information from roughly half the simulations, and so corresponding averages and deviations for the mass $M_{\rm irr}$ and spin $j_{\rm AH}$ only include those.

	Mergers		Scatters	
	Average	Maximum deviation	Average	Maximum deviation
$E_{\rm rad}/M$	0.295	2.3%	0.252	2.2%
$E_{\rm phys}/M$	0.265	2.6%	0.222	2.1%
$J_{\rm rad}/J$	0.643	2.6%	0.580	1.2%
$J_{\rm phys}$	0.605	5.2%	0.531	0.7%
$M_{\rm irr}/M$	0.607	0.3%		
$j_{\rm fin}$	0.869	3.2%		
$j_{\rm QNM}$	0.890	4.4%		
$j_{\rm AH}$	0.889	2.2%		

B. Computational grid and uncertainties

We have evolved all binary configurations on a numerical grid consisting of ten nested refinement levels, three levels with one component centered on the coordinate origin and seven levels with two components each, centered on either BH. Using the notation of Sec. II E of Ref. [14], the exact grid setup in units of M (rounded to three significant digits) is given by

$$\{(258, 184, 92)(13.8, 6.90, 3.45, 1.73, 0.863, 0.431, 0.216), h\}.$$

Our standard resolution is $h = M/223$, but for convergence testing we have also evolved one merger case using a coarser resolution $h_{\rm c} = M/195$ and finer resolution $h_{\rm f} = M/250$. GWs have been extracted on a set of six concentric spheres of coordinate radii $R_{\rm ex} = 57.5M$ to $86.3M$ in steps of $5.76M$.

The convergence analysis for the recoil velocity is shown in Fig. 2. Here we define a *time-dependent kick* as the quotient of the radiated momentum and the final BH mass: $v_{\rm kick} = -P_{\rm rad}(t)/M_{\rm fin}$. The figure demonstrates second-order convergence. Richardson extrapolation reveals a relative uncertainty of the numerical kick velocity obtained with medium resolution of about 9%.

A second main source of error is inherited from the use of finite extraction radii. We study the resulting error by analyzing $v_{\rm kick}(t)$ extracted for the high-resolution simulation of the test model at six different radii in Fig. 3. For this purpose we have first aligned the velocity functions in time to compensate for differences in the propagation time, and fitted the resulting curves with either of

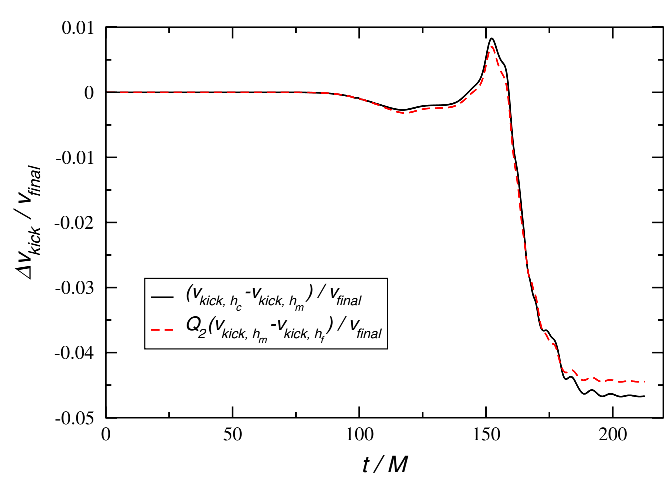

FIG. 2 (color online). Convergence test for the recoil velocity $v_{\rm kick}(t) \equiv P_{\rm rad}(t)/M_{\rm fin}$. The convergence factor $Q_2 = 1.459$ corresponds to second-order convergence.

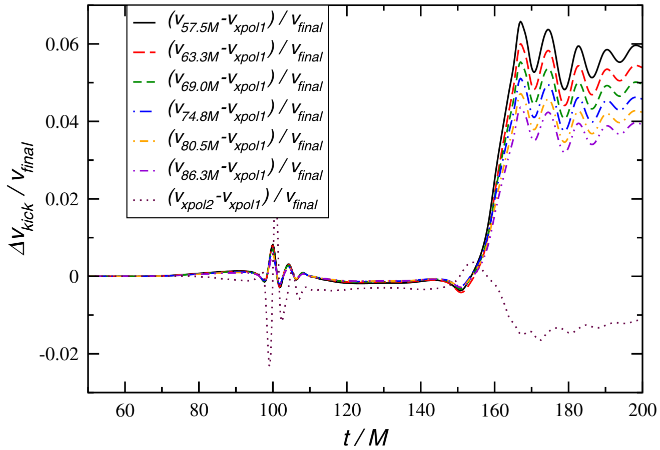

FIG. 3 (color online). Difference of the recoil velocity $v_{\rm kick}(t) \equiv P(t)/M$ at different extraction radii from the reference curve obtained by extrapolation according to Eq. (3), referred to as "xpol1". We also show the difference of a second fit assuming a quadratic term according to Eq. (4), referred to as "xpol2".

$$v(t, r_{\rm ex}) = v_0(t) + \frac{v_1(t)}{r_{\rm ex}}, \tag{3}$$

$$v(t, r_{\rm ex}) = v_0(t) + \frac{v_1(t)}{r_{\rm ex}} + \frac{v_2(t)}{r_{\rm ex}^2}. \tag{4}$$

The predicted recoil for infinite extraction radius is given by $v_0(t)$.

The fractional error in the velocity, as inferred from the difference between the largest extraction radius used in practice ($r_{\rm ex} = 86.3M$) and the extrapolated value, is roughly 4%. We note that the main contributions to the error in the velocity typically are opposite in sign: finite resolution truncation error causes an underestimate of the recoil, while the use of finite extraction radius results in an overestimation.

In the remainder of this paper we report radiated quantities obtained at $r_{\rm ex} = 86.3M$ and at medium resolution and cite a combined error due to discretization and finite extraction radius of 13%. For the reasons mentioned above, we consider this a rather conservative estimate of the uncertainties.

In order to calculate the physical radiated momenta and energy (the quantities with a subscript "phys" in Table I), we exclude from the extraction the early part of the gravitational waveforms up to $t - r_{\rm ex} = 50M$, which is dominated by spurious radiation due to the initial data. For reference, the total radiated quantities that include the spurious radiation are also shown in the table with a subscript "rad".

Before we discuss our results in more detail, we conclude this section with a summary of further diagnostic quantities. The total center-of-mass energy of the system is given by the ADM mass of the initial data as provided by the spectral solver. The radiated momenta and energy enable us to calculate the final BH mass

$$M_{\rm fin} = M - E_{\rm rad}. \tag{5}$$

In the case of scattering configurations this mass is to be interpreted as the sum of the individual BH masses in the limit of large separation. Balance arguments further provide us with an estimate for the dimensionless spin of the merged BH,

$$j_{\rm fin} = \frac{L - J_{\rm rad}}{M_{\rm fin}^2}. \tag{6}$$

By virtue of the symmetry of the binaries studied in this work, the angular momentum of the BH binary as well as that contained in the gravitational radiation points in the z direction, defined as the direction of the initial orbital angular momentum.

We can also estimate the spin $j_{\rm QNM}$ of the final BH by fitting the gravitational waveform at late-times with an exponentially damped sinusoid. The quasinormal mode (QNM) frequency and damping time of this signal can be inverted to obtain $j_{\rm QNM}$ (see e.g. Refs. [32,33]).

An alternative measure for the final spin is given in terms of the irreducible mass of the apparent horizon. For this purpose we rewrite Christodoulou's relation (1) for the post-merger BH as

$$j_{\rm AH} = 2\frac{M_{\rm irr}}{M_{\rm fin}}\sqrt{1 - \frac{M_{\rm irr}^2}{M_{\rm fin}^2}}. \tag{7}$$

III. RESULTS

In this section we describe the main results from our study. In Sec. III A we first discuss the total *energy* radiated, before turning to the question of net *momentum* in Sec. III B. In Sec. III C we comment on the relationship between these quantities, borrowing results from a wider set of published simulation results, noting that in magnitude the ratio of these two quantities is nearly constant. Based on this observation, we describe several speculative extrapolations to guess what the "ultimate" kick might be in Sec. III D. Finally, in Sec. III E we comment on what our results imply about the mechanism of the antikick.

A. Radiated energy

Before we discuss in detail the radiation of linear momentum, we consider the energy carried away in the form of gravitational radiation. Radiated energies are listed in the rows labeled $E_{\rm rad}$ and $E_{\rm phys}$ in Table I; again, the latter row excludes contributions due to spurious radiation inherent in the initial data.

Note that the radiated energy shows little variation (the "maximum deviation" column) within either sequence. Also, the orientation of the spins in the xy plane has no impact on the outcome (merger or scattering) of the binary interaction. This confirms the observation made for the astrophysically more relevant case of quasicircular

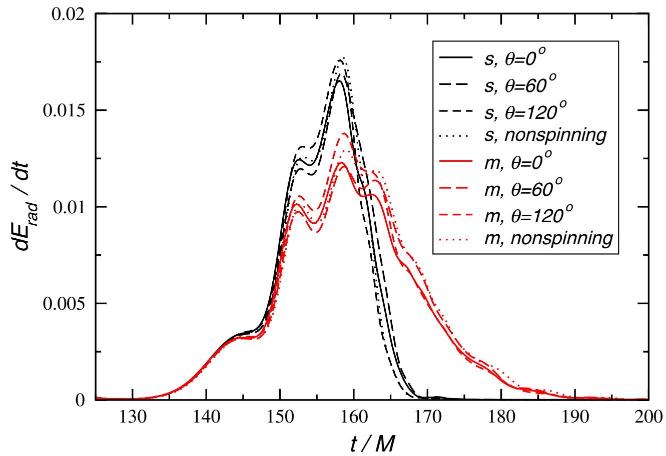

FIG. 4 (color online). Radiated energy flux $dE_{\rm rad}/dt$ for a selected set of initial phase angles from both sequences (s for scatter, m for merger), and including the nonspinning cases.

superkick configurations, as discussed, for example, in Sec. III B of Ref. [3]: the spin orientation in the orbital plane does not significantly influence the dynamics *within* the orbital plane. Furthermore, the spin *magnitude* makes little difference, as evidenced in that the maximum deviation listed includes the nonspinning cases. Figure 4 shows the energy flux $dE_{\rm rad}/dt$ for a few cases from the two sequences.

The energy flux for all models within a sequence has similar levels of agreement, so we restrict the number of curves in the figure for clarity.

In summary, the total radiated energy is essentially independent of the orientation of the spins or, indeed, the presence of the spins in the first place.

B. Gravitational recoil

The two sequences studied in this work either result in a merger or in a scattering where no common apparent horizon forms and the BHs fly apart until they can be regarded as isolated. For merging binaries the total recoil is defined in the traditional manner: the linear momentum radiated in the form of GWs has to be balanced by the recoil of the post-merger BH. For scattering runs, we similarly define a *total kick* of the binary system, but now the momentum due to the recoil is distributed over two individual BHs instead of one. By virtue of the symmetry of all configurations studied in this work, the two BHs acquire equal linear momentum after scattering, i.e., they move in the z direction with identical velocities.

The final recoil velocity is shown in the top panel of Fig. 5 for all parameters. The data exhibit the same sinusoidal dependence of the recoil on the orientation angle θ that was found for astrophysical binaries. These best-fit sinusoids, also shown in the figure, are

$$v_{\rm kick,s} = 12\,200 \cos(\theta - 2.53)\ {\rm km/s}, \tag{8}$$

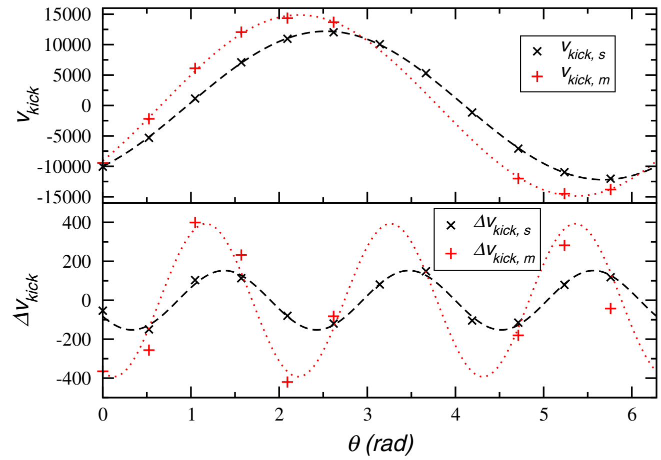

FIG. 5 (color online). Top panel: the final recoil velocity as a function of the spin orientation θ for the m- and the s-sequence. The curves are best-fit sinusoids to the data. Bottom panel: subdominant contribution to the final recoil velocity, obtained by subtracting from the total recoil the sinusoidal fit in Eq. (8).

$$v_{\rm kick,m} = 14\,900 \cos(\theta - 2.23)\ {\rm km/s}. \tag{9}$$

Most significantly, the magnitude of the total radiated linear momentum is quite similar between the two cases. It is also interesting to note, however, that both the maximum recoil and the radiated energy are a bit larger for the m-sequence than for the s-sequence. We will investigate this feature more closely in the following section.

Boyle, Kesden, and Nissanke [11,12] systematically expanded the mass, recoil velocity, and spin of the remnant BH resulting from a binary BH merger in terms of the binary's mass ratio q and of the individual spins χ_i of the two BHs. For equal-mass "superkick" configurations where the two BHs have the same Kerr spin parameter ($\chi_1 = \chi_2 = \chi$) their result for the final kick can be expressed in the form

$$v_{\rm kick} \simeq k_1 \chi \cos(\theta - \theta_1) + k_3 \chi^3 \cos(3\theta - \theta_3) + \mathcal{O}(\chi^5). \tag{10}$$

The bottom panel of Fig. 5 shows that the subdominant contribution to the kick is indeed well fitted by a function of this form for *both* the merging and scattering sequences, whereas the spin expansion of Refs. [11,12] considered mergers only. A fit of the data including third-order contributions in χ yields

$$k_1^{(m)} = 24\,119, \qquad k_3^{(m)} = -1722, \qquad \theta_1^{(m)} = 2.227, \qquad \theta_3^{(m)} = 0.373 \tag{11}$$

for merging binaries, and

$$k_1^{(s)} = 19\,634, \qquad k_3^{(s)} = -636, \qquad \theta_1^{(s)} = 2.532, \qquad \theta_3^{(s)} = 0.991 \tag{12}$$

in the scattering case. For comparison, the astrophysical mergers of superkick configurations lead to $k_1^{(J)} = 3769$,

TABLE II. Summary of the initial dimensionless spin magnitude, radiated energy, and maximum recoil velocity. For the latter quantity, we have two estimates, the first ($v_{\max,1}$) obtained using the leading-order term in (10), the other ($v_{\max,3}$) also including the next-to-leading-order term.

Reference	χ	$E_{\rm phys}/M$	$v_{\max,1}$(km/s)	$v_{\max,3}$(km/s)	$M(v_{\max,1}/c)/E_{\rm phys}$	$M(v_{\max,3}/c)/E_{\rm phys}$
QC [34]	0.2 . . . 0.9	0.0401	3682	3680	0.306	0.306
H [13]	0.8	0.128	11 988		0.312	
s-sequence	0.621	0.220	19 634	19 043	0.298	0.289
m-sequence	0.621	0.265	24 119	22 398	0.303	0.282

$k_3^{(J)} = 228$ for the simulations by the Jena group [3] and to $k_1^{(R)} = 3622$, $k_3^{(R)} = 216$ for the simulations by the Rochester group [5] (cf. Table V of Ref. [12]). Considering that these simulations were carried out by independent codes and considering different values of the individual BH spins, these last sets of numbers are in remarkable agreement and within the numerical errors.

Let us assume that the "true" values of the parameters for astrophysical binaries are given by an average of the Rochester and Jena results: $k_1 = 3695$, $k_3 = 222$. Then we find that, within the accuracy of the numerics, the fitting coefficients of different orders in our relativistic mergers and those in "astrophysical" mergers are roughly consistent with a single proportionality relation: $|k_1^{(m)}/k_1| \simeq 6.5$ and $|k_3^{(m)}/k_3| \simeq 7.8$.

As discussed below, these findings may have interesting implications to estimate the maximum kick achievable in any binary BH encounter.

C. The relation between energy and recoil

Let us now investigate the relation between the radiated energy $E_{\rm phys}$ and the maximum[1] recoil velocity $v_{\rm kick}$. We will use this relation in the next section to extrapolate existing information on BH binaries and roughly estimate the ultimate recoil achievable in any ultrarelativistic BH encounter.

The search for a trend between radiated energy and recoil velocity requires use of data from different configurations. We here consider data obtained by Healy *et al.* [13] for hyperbolic encounters (H in Table II) and the simulations of Lousto and Zlochower [34] for approximately quasicircular binaries (QC in Table II). In order to assess the impact of subdominant contributions to the expansion in Eq. (10), we list in this table two values for the maximum recoil velocity: (i) the leading-order prediction $v_{\max,1}$ using only the linear term and (ii) the value $v_{\max,3}$ obtained by also including the cubic term. The table further shows the radiated energy $E_{\rm phys}$ and the value (or range) of the spin magnitude χ of the individual BHs considered. For either estimate of the maximum kick velocity, we have calculated the ratio to the radiated energy.

Some comments on this table are in order. First, in contrast with high-energy collisions, BH binaries in quasicircular orbits radiate a significant fraction of their energy during the early inspiral phase. For example, the total radiated energy for a nonspinning equal-mass binary inspiral and merger is approximately $0.05M$ [35], out of which about 70% ($0.035M$) is radiated in the last two orbits; see e.g. [14,36–38]. The radiated energy for the quasicircular entry in Table II has been obtained by averaging the values reported by Lousto and Zlochower for their $\chi = 0.9$ sequence, but we cannot rule out that the relevant value may be larger. On the other hand, the early inspiral appears to have less impact on the accumulated recoil, as intuitively expected: the GW flux increases more slowly during early inspiral, so that the orbital average of the net momentum flux is closer to zero (cf. Sec. III E of [3]). In the remainder of this section, we will employ the values listed in the table, but one should keep in mind the above caveats and lack of high-precision data for long quasicircular inspirals.

A second comment on Table II concerns the study by Healy *et al.* [13]. The sequence they simulated differs from the other studies in that they vary the linear momentum of the BHs and, thus, the kinetic energy of the binary. Because of this, we only use one simulation from their Fig. 2, which provides a maximum kick. In units of the initial BH rest mass $M_{\rm rest}$, they report an initial linear momentum of $P/M_{\rm rest} = 0.308$, a radiated energy of 15%, and $v_{\rm kick} = 9590$ km/s for this simulation. In order to use their data, we need to adjust for the different normalization (with respect to the BH rest mass in Ref. [13] and with respect to the total center-of-mass energy in our comparison). Including linear momentum, the dynamical BH mass is a factor 1.174 larger than the rest mass. Therefore we estimate the energy radiated in their binary system as 12.8% of the total energy of the system. Finally, their data only allow us to estimate the maximum kick using the leading-order extrapolation in the spin magnitude χ.

In the final two columns of Table II we show the ratio of the maximum kick velocity, extrapolated to maximal spin $\chi = 1$, to the radiated energy per center-of-mass energy of

[1] "Maximum" in this context refers to variation over the orientation angle θ while keeping the center-of-mass energy constant. To avoid confusion, we will refer to the "absolute maximum" achievable when we vary also the intrinsic parameters of the binary (center-of-mass energy, mass ratio, and spin magnitudes) as the "ultimate" kick.

the binary. In this column we measure the recoil velocity in units of the speed of light c. The results suggest that, to leading order and for equal-mass binaries, the maximum of the kick as we vary the orientation angle θ is proportional to the radiated energy, which itself is to leading order independent of θ and approximately equal to the energy radiated by the binary's nonspinning counterpart.

D. Conjectures on the ultimate kick

In this section, we speculate on several possible ways of extrapolating our results to the ultimate recoil achievable in any binary black hole encounter and on the related uncertainties.

Assuming that the scaling observed in the previous section remains valid for arbitrary center-of-mass energies, we could derive a leading-order estimate of the maximum kick possible in any BH binary encounter *if we knew the maximum radiated energy for equal-mass, nonspinning binaries*. This maximum energy has as yet not been determined, but ultrarelativistic grazing collisions with boost factor $\gamma \approx 3$ have been found to radiate as much as $35 \pm 5\%$ of the total energy of the system [39]. Extrapolation of the energy radiated by equal-mass binaries has so far only been achieved for head-on collisions. In this case, Ref. [40] predicts an increase in $E_{\rm phys}$ by a factor of about 1.6 as γ increases from 3 to infinity via extrapolation. Based on this information, let us *assume* as a working hypothesis, that grazing collisions of equal-mass, nonspinning BHs will result in maximum energies up to about 50% of the total mass. When combined with the findings in Table II, this would result in an ''ultimate kick'' of about 0.15 times the speed of light, or $\sim$45 000 km/s. In view of the assumptions made for this derivation, this prediction should only be regarded as a rough estimate.

The most uncertain assumption in the previous analysis concerns the maximum energy that can be radiated by equal-mass, nonspinning binaries. It may well be possible that *all* the excess kinetic energy can be radiated as we fine-tune the impact parameter around threshold. Let us assume for the sake of argument that this conjecture is true. Then, in the *large-γ limit*, the maximum energy radiated would be $\sim$100% for finely-tuned initial data. However, since spin does not change with boost, most of this energy will be radiated when the spin is insignificant relative to the total mass. The relevant question then becomes: *how much excess kinetic energy is left once the spin becomes significant?* Let us postulate that, for a given γ, the kick is proportional to the relative excess kinetic energy multiplied by an ''effective'' dimensionless spin $\chi_{\rm eff}$,

$$v \propto \frac{\gamma - 1}{\gamma}\chi_{\rm eff}. \tag{13}$$

If we were to let $\chi_{\rm eff} = \chi$, we would clearly have a problem, as we would not be accounting for the fact that the spin becomes unimportant for large γ, and we would arrive at a maximum kick speed at $\gamma = \infty$. If instead we define $\chi_{\rm eff}$ as the spin angular momentum normalized by the *total* mass, not the rest-mass, $\chi_{\rm eff} \equiv S/M^2 = \chi m^2/(\gamma m)^2 = \chi/\gamma^2$, then

$$v = v_0 \frac{\gamma - 1}{\gamma^3}\chi, \tag{14}$$

where v_0 is the proportionality constant. This scaling takes into account the two limiting cases: for $\gamma = 1$ no kinetic energy is radiated, so there should be no kick; for $\gamma \to \infty$ the spin is irrelevant, so (again) there should not be a kick. Quite interestingly, this *ad hoc* ansatz yields a maximum kick at $\gamma = 3/2$, the case studied here, and hence we can use these results to calculate v_0: this predicts a maximum kick of $\approx$ 24 000 km/s. A higher γ binary, sufficiently fine-tuned to the threshold of merger, would presumably also yield a comparable maximum kick. For then it would lose most of its excess kinetic energy to gravitational radiation, and hence the last few orbits (that contribute most to the kick) prior to merger or scatter will occur at a much lower γ.

A potentially more accurate way to estimate the maximum kick could employ the Boyle-Kesden expansion, Eq. (10). It is plausible to assume that the coefficients k_1 and k_3 appearing in the fitting formula should only be functions of γ. The numerical estimate of these coefficients shows that $|k_1^{(m)}/k_1| \simeq 6.5$ and $|k_3^{(m)}/k_3| \simeq 7.8$, so these coefficients could have (to leading order) the *same* functional dependence on γ: say, $k_1 = \alpha k_3 = f(\gamma)$, where α is a constant. The main difficulty here consists of the fact that it is impossible to determine this functional dependence having ''sampled'' the function at only two values of γ (i.e., $\gamma = 1$ and $\gamma = 1.52$). If we assume $f(\gamma) \sim \gamma^c$ we would get that $c \approx 4$ is a good power-law scaling in the range $1 \le \gamma < 2$. However it would be perfectly legitimate to assume (say) that $f(\gamma) = a + b\gamma^c$, and then it would be impossible to constrain the three parameters of the model with two data points. In fact, we know that $f(\gamma)$ must asymptote to a finite limit as $\gamma \to \infty$, because the kick velocity is limited by the speed of light.

In summary, with presently available data, we cannot discriminate between the scenarios discussed in this section: (i) a tentative guess of 45 000 km/s if the proportionality between maximal kick and radiated energy holds for arbitrary values of the boost parameter; (ii) a significantly lower velocity of $\sim$24 000 km/s if the spin remains insignificant for the binary dynamics until most of the kinetic energy has been radiated away; or (iii) a more complex scenario where the ultimate kick can only be predicted by a systematic expansion, as proposed by Boyle, Kesden, and Nissanke. This uncertainty provides further motivation to simulate sequences of merging binaries with larger γ factors.

E. Time dependence of P_{rad} and the antikick

Up until now, we have exclusively analyzed the final value of the radiated linear momentum $P_{\mathrm{rad}}(t = \infty)$, but not its time evolution. Here, we should distinguish between "local" BH dynamics and the presence of local extrema in $P_{\mathrm{rad}}(t)$, as measured in the wave zone. The former has been studied, for example, in Refs. [7,8], and reveals substantial reversals (of order 10^3 km/s) in local estimates of the BH velocities. These large variations are not mirrored in the radiated linear momentum as measured far away from the BHs,[2] and henceforth with "antikick" we exclusively refer to the deceleration visible in P_{rad} as measured far away from the BHs.

This antikick has been discussed in connection with ringdown radiation from the remnant BH in Refs. [42–44]. The addition of ringdown to post-Newtonian estimates of the recoil generated by unequal-mass, nonspinning binaries has resulted in excellent agreement with numerical calculations. Reference [10] attributed the antikick to deformations of the common horizon after merger.

On the other hand, it is unclear if the antikick has any physical significance at all. Consider the instantaneous momentum radiated in GWs in any specific direction as given approximately by a sinusoidal function (from orbital motion and then ringdown if a merger occurs), modulated by an envelope proportional to the radiated energy, which is initially increasing (motion to closest approach) and then decreasing (from ringdown in a merger scenario or due to increasing separations in a scattering scenario). There is no *a priori* reason to expect that this function, integrated in time from $t = -\infty$ to some $t = t_f$, should generically have its extremum at $t_f = \infty$. As with many other properties related to kicks, the difference between the maximum and the final value depends on the *phase* of the modulated sinusoid and it does not require any new physical mechanism to explain it.

The one piece of evidence we can provide to the explanation of the antikick can be seen in Fig. 6, where we plot the radiated linear momentum, converted into a recoil velocity by rescaling with the final mass of the system, as a function of time. Antikicks (a decrease in the absolute magnitude of the kick velocity as a function of time) are clearly present in both merger and scattering cases. Hence, common horizon deformations cannot be "the" explanation, as a common horizon does not form in the scattering cases. Furthermore, since the magnitude of the antikick clearly depends on the initial phase of the binary, the data are consistent with the modulated-sinusoid description given in the previous paragraph.

[2]These large local velocities are consistent with the superkick explanation in Ref. [41], as such velocities are necessary to produce sufficient blue- or redshifts of the radiated momentum to account for the actual kick.

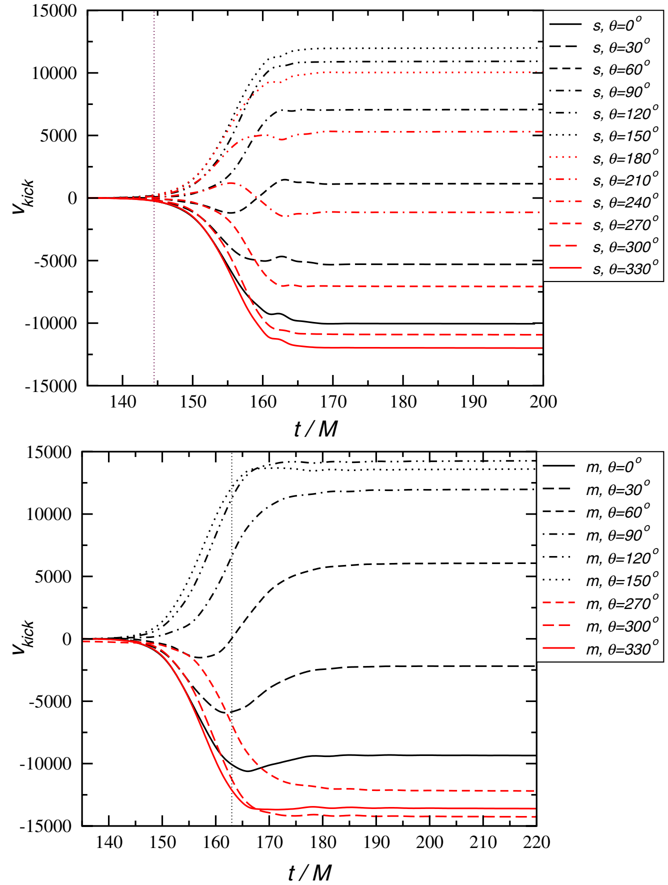

FIG. 6 (color online). The recoil v_{kick} as a function of time extracted at $r_{\mathrm{ex}} = 86.2M$ for the s-sequence (upper panel) and the m-sequence (lower panel). For reference, the vertical lines mark the minimum coordinate separation of the punctures (for the scattering runs) and the common apparent horizon formation (for the mergers).

IV. CONCLUSIONS

In this work we have studied grazing collisions of equal-mass BH binaries with antialigned spin angular momenta in the orbital plane—the so-called superkick configurations. We have studied two sequences with a moderate boost of $\gamma \approx 1.5$: an m-sequence leading to formation of a common apparent horizon; and an s-sequence where the two BHs eventually scatter off to infinity. Within each sequence, we have varied the phase angle of the spin angular momentum relative to the x-axis connecting the initial BH positions. For comparison, we have also included two simulations of equal-mass, nonspinning BHs with impact parameters corresponding to the two sequences.

We have shown that the qualitative details of the recoil (sinusoidal dependence on phase, and the maximum being proportional to the total energy radiated) is independent of whether merger or scattering occurs. For the phase angle resulting in the maximum recoil, we find $v_{\mathrm{kick}} = 14\,900$ km/s in the merger sequence, and $v_{\mathrm{kick}} = 12\,200$ km/s for the scattering case. Thus, the mechanism responsible for the superkick is, to leading order, not related to merger dynamics. We have further found, as with earlier observations for approximately quasicircular binary systems, that the dynamics and net energy radiated in the orbital plane are essentially unaffected by the presence or orientation of the spins. We obtain radiated energies of about $(22.2 \pm 2.2)\%$ and $(26.5 \pm 2.7)\%$ of the total center-of-mass energy for the s- and m-sequence, respectively, with no dependence on the orientation of the spins to within the quoted uncertainties of these numbers. The corresponding nonspinning binaries also lead to similar radiated energies to within the quoted uncertainties.

The ratio between the maximum recoil and the radiated energy for our two sequences is very similar, and this has led us to compare our data to available results from the literature on quasicircular binaries and hyperbolic encounters of equal-mass, spinning binaries with opposite spins in the orbital plane. Similar ratios between maximum radiated energy and recoil velocity are found in all cases. Assuming that this scaling holds for arbitrary γ, a *rough* guesstimate for the ultimate kick would be around 45 000 km/s. On the other hand, it is equally possible that the spin of the individual holes is insignificant until most of the kinetic energy has been radiated away. We have estimated the resulting consequences by introducing an *effective spin parameter*. The ultimate kick would be smaller in this case, about 24 000 km/s. Even if neither of these simplifying assumptions turn out to be valid, the determination of the ultimate kick can be obtained from a systematic expansion of the kick dependence on the binary parameters, as proposed by Boyle, Kesden, and Nissanke [11,12]. In view of the substantial number of additional simulations required for this purpose, we postpone such a study to future work.

Finally, we have analyzed the time evolution of the radiated linear momentum with regard to the presence or absence of local extrema. The decrease in the absolute magnitude of the recoil velocity after reaching a local extremum has been dubbed antikick. In both merging and scattering sequences, we observe antikicks, though the particular value is dependent on the initial phase. Therefore, as with the superkick, antikicks are not a property exclusively related to the formation and subsequent evolution of a common horizon.

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